Broadband

Data Sheet

Telewest Communications is a leading broadband cable communications operator that provides multi-channel television, telephone and Internet services to millions of UK houses. It also provides voice and data telecommunications services to over 60 000 business customers.

In March 2000 Telewest Communications launched Blueyonder, the UK's first high speed broadband Internet service for home personal computers. Since then the number of broadband connections in the UK has grown rapidly.

The table below shows this growth over a 3 year period from January 2001 to January 2004.

y represents an estimate of the total number of broadband connections in thousands and *x* represents the number of 6 month time intervals since January 2001.

Time (6 mth intervals) x = 0 on 1st January 2001	**No. of Connections (thousands)**
x	y
0	110
1	200
2	370
3	830
4	1500
5	2370
6	3200

Broadband

Worksheet

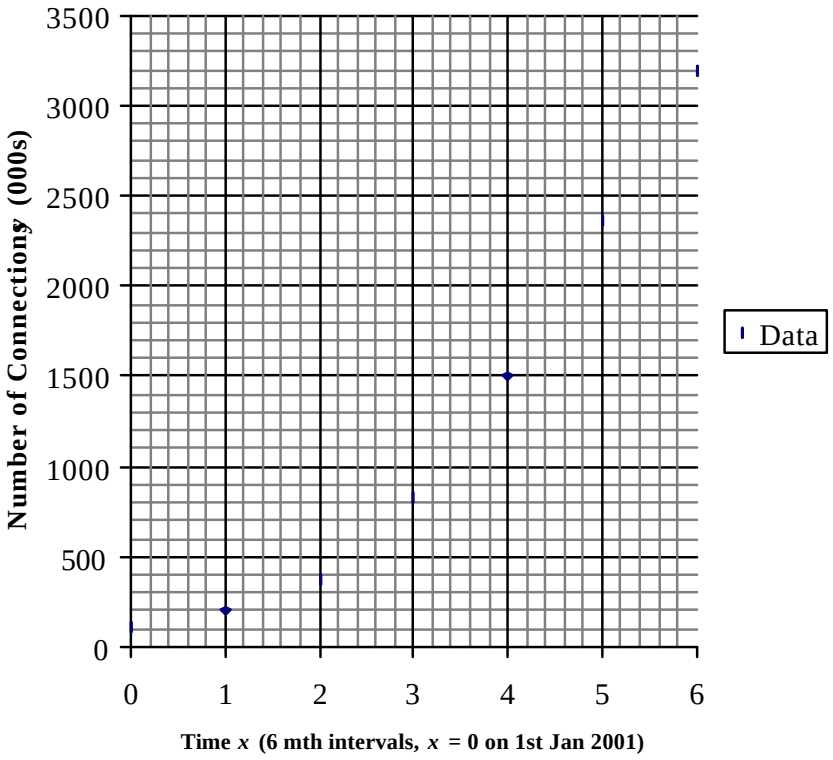
In this activity you will find a quadratic function to model the data given on the Data Sheet.

Enter the data into an Excel spreadsheet and use the Chart Wizard to **draw a graph**.

Step 1 Left click on **XY(Scatter)** and choose the **1st option** to give a scatter graph with data points, but no lines.
Step 2 Left click on **Series** and enter 'Data' in the **Name** box.
Step 3 Enter the **Titles** as shown on the graph below and choose to have both **Major** and **Minor Gridlines**.
Step 4 Choose to have the Chart as an **object in the worksheet** alongside the data.

Format the plot area, gridlines, labels etc so your graph looks similar to that shown below.

Growth in UK Broadband Connections



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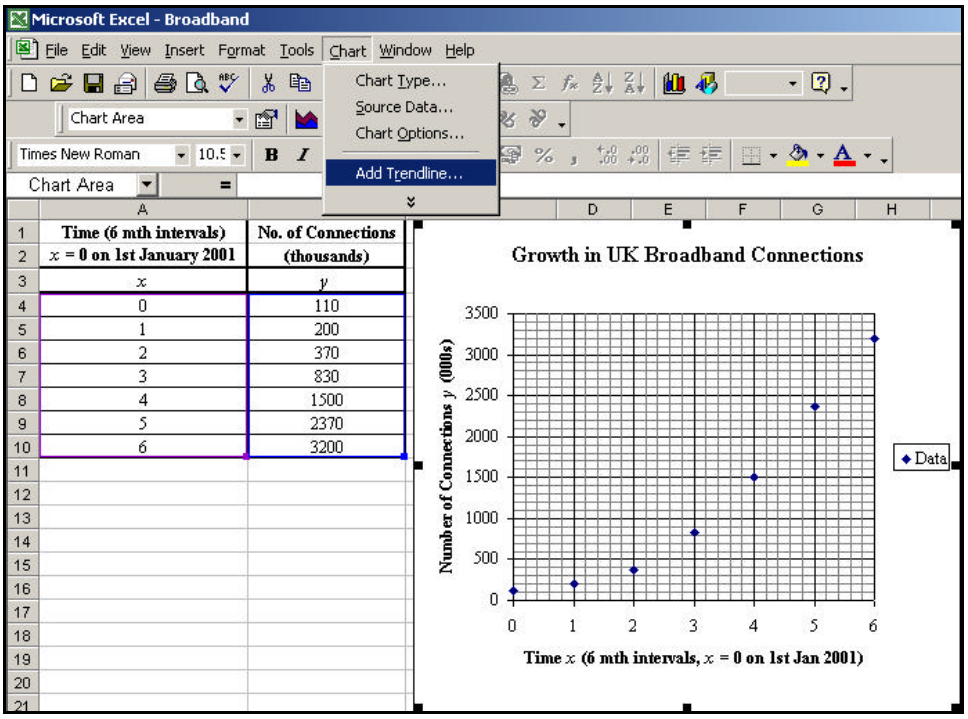
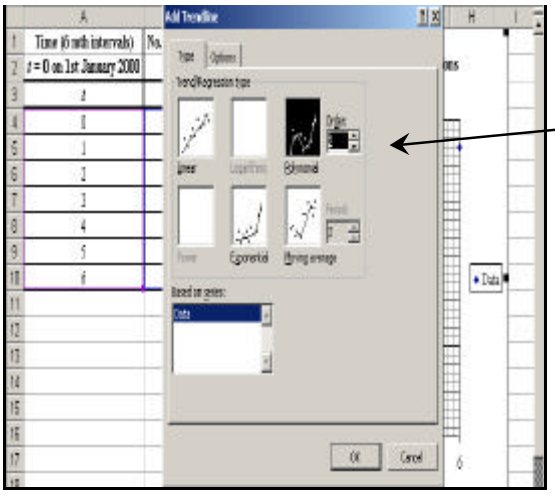
To find a quadratic function to model the data, proceed as follows:

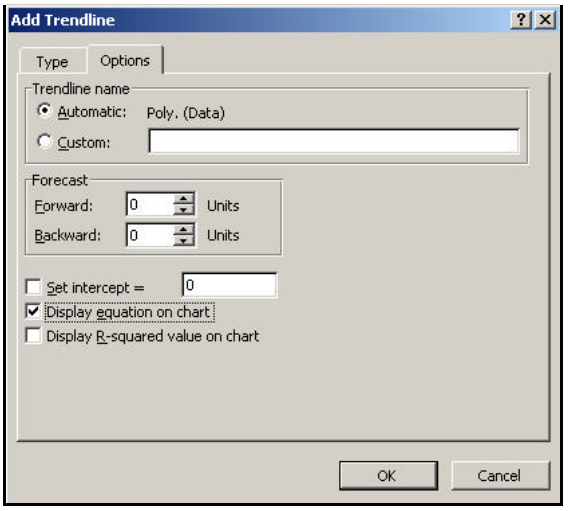
Left click on **Add Trendline** in the **Chart** menu.





Left click **Polynomial**. **Order 2**, means that Excel will give the best quadratic function to model the data.

In the **Options** menu, left click the **Display equation on chart** box.



Then left click **OK**.

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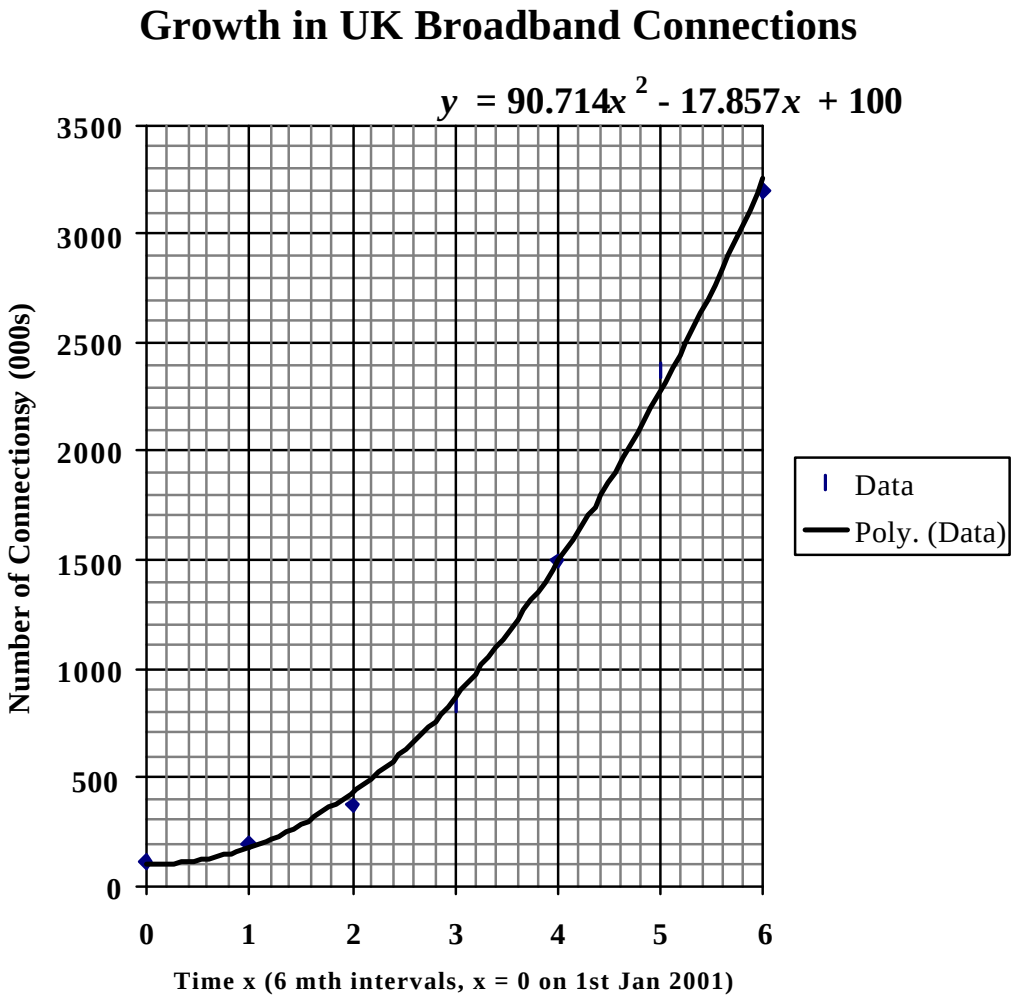
Excel will draw a trendline on the graph and give its equation.



Excel has given the best quadratic model for the data to be $y = 90.714x^2 - 17.857x + 100$

You may wonder if the model given by Excel would still be acceptable if the coefficients were rounded to more convenient values such as $y = 91x^2 - 18x + 100$

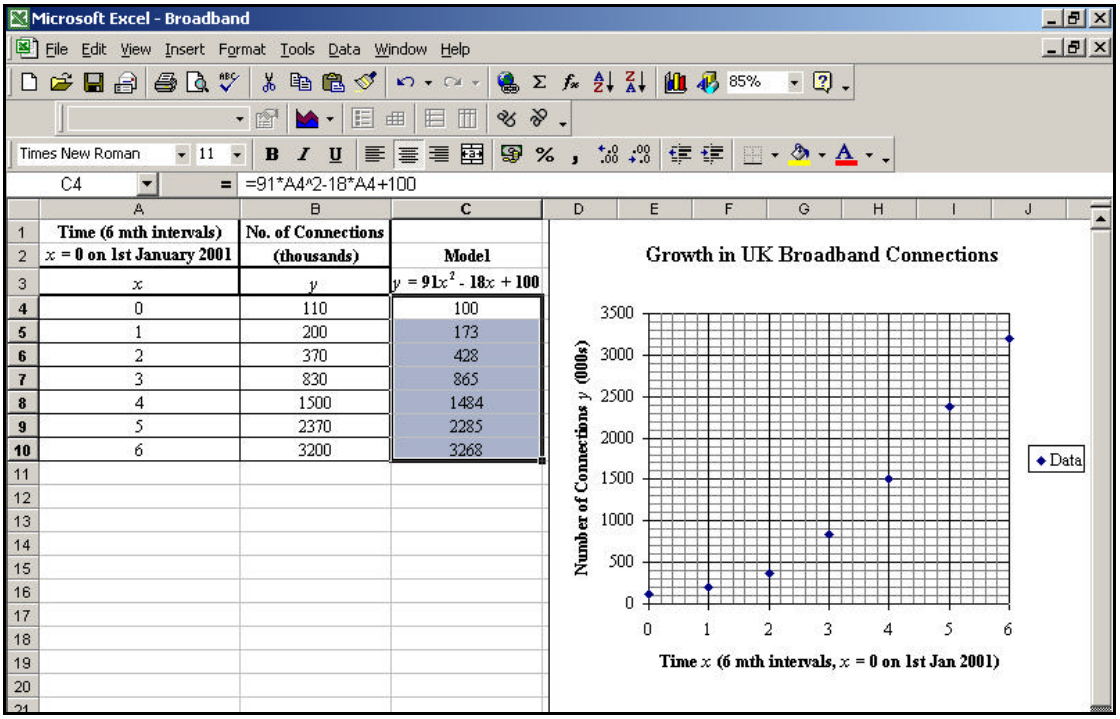
On other occasions you may wish to check a model that you have found yourself (perhaps by assuming that the model is of the form $y = ax^2 + bx + c$ and then finding values for a, b and c by substituting the co-ordinates of 3 data points to give a set of simultaneous equations).

The rest of this activity shows how to **use Excel to check a model you enter yourself**.

Remove the trendline by left clicking on it and then pressing the **Delete** key.

Put the headings shown below in cells C2 and C3.

Then enter the formula '**=91*A4^2-18*A4+100**' in cell C4 and use **fill down** to give the values shown below in cells C4 to C10.



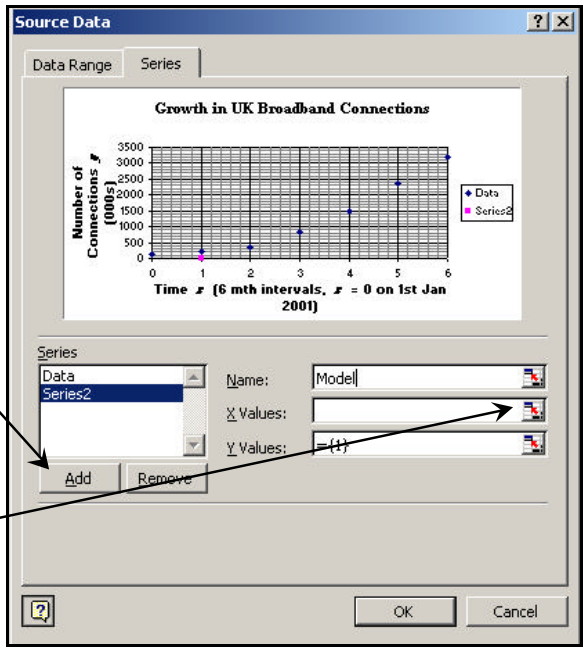
	A	B	C
1	Time (6 mth intervals)	No. of Connections	
2	x = 0 on 1st January 2001	(thousands)	Model
3	x	y	$y = 91x^2 - 18x + 100$
4	0	110	100
5	1	200	173
6	2	370	428
7	3	830	865
8	4	1500	1484
9	5	2370	2285
10	6	3200	3268

Right click on the chart area then left click on **Source Data** (or select **Source Data** from the **Chart** menu).

In the **Series** menu left click **Add** to get the menu shown here.

In this menu enter the **Name** 'Model' for Series 2.

To select the x values for Series 2, left click on the button at the right hand end of the **X Values** box.



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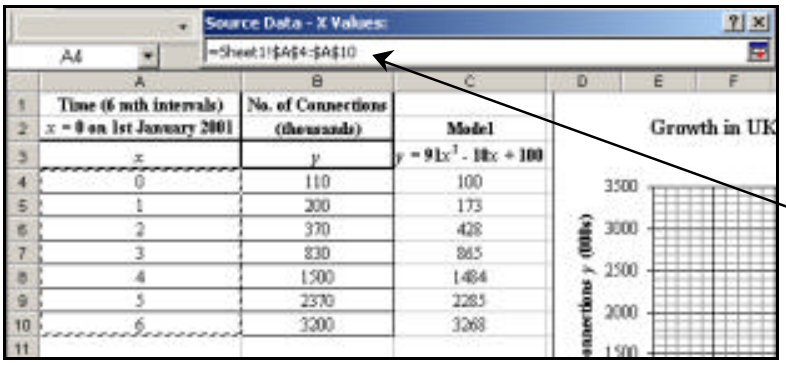


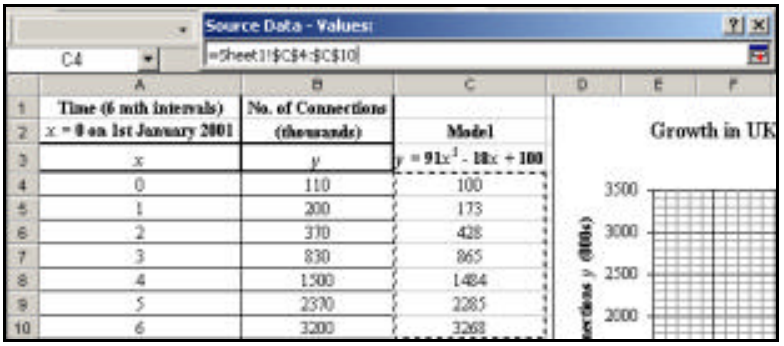
Excel will take you back to the data.

Select the *x* values in cells A4 to A10
The cell references will appear in the box.

Now left click the button at the right hand end of the box.

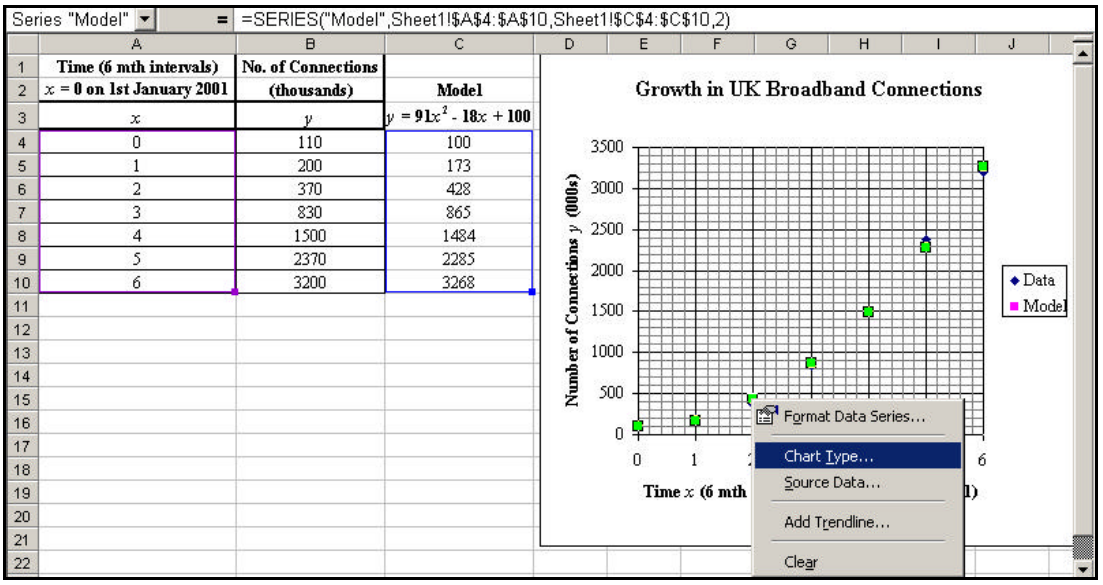
To select the *y* values for Series 2, left click on the button at the right hand end of the **Y Values** box. Again Excel will take you back to the data.

Select the *y* values in cells C4 to C10. Again the cell references will appear in the box.



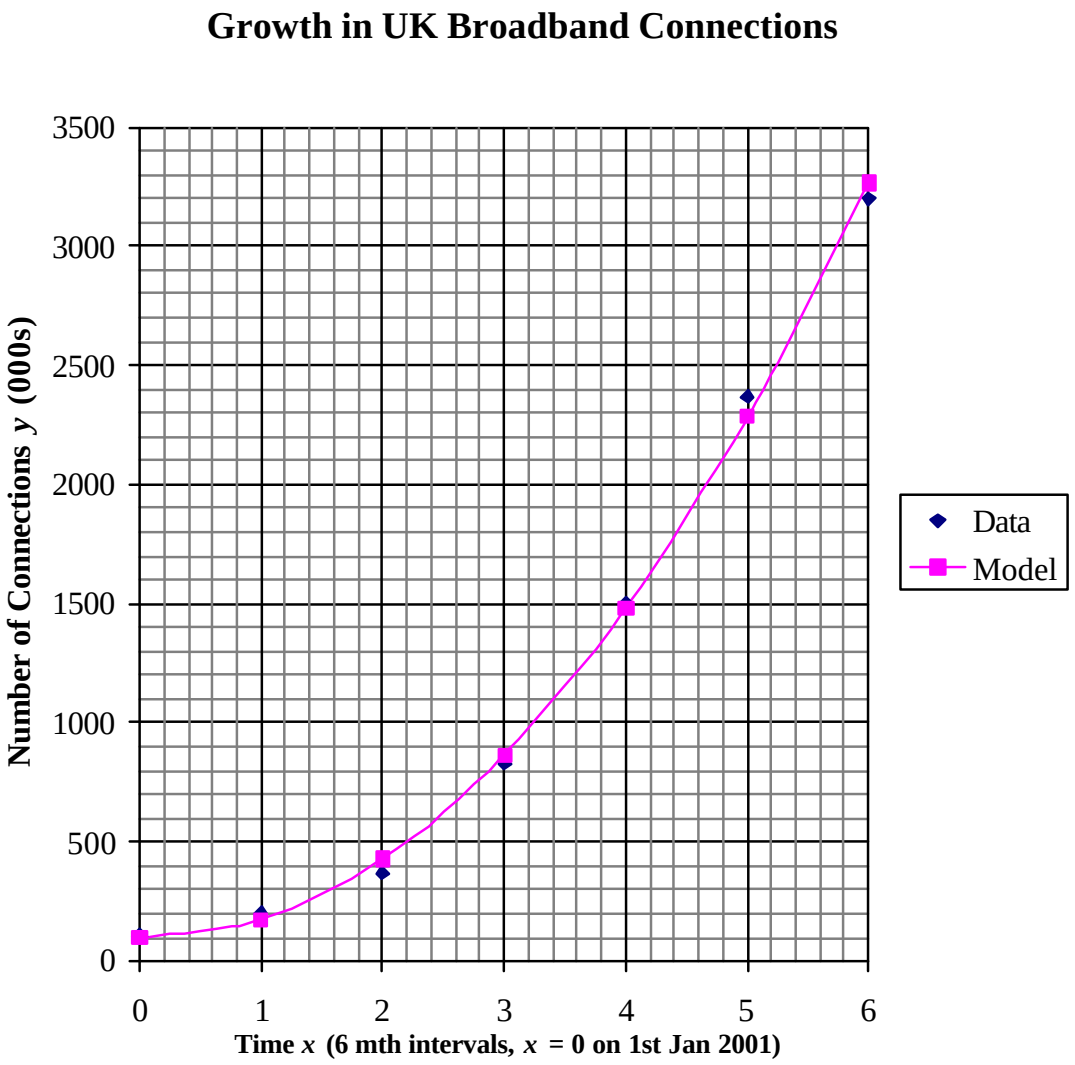
Left click the button at the right hand end of the box, then OK, and a second set of points representing the model will appear on the graph.

Right click on one of the new points to get the menu shown below, then left click on **Chart Type**.



Choose **the 2nd type of scatter graph** to give data points connected by smoothed lines. Then left click **OK**.

The graph should now appear as shown below.



Compare this graph with that shown on page 4. Rounding the coefficients to the nearest whole number has not seriously affected how well the model fits the data.

If you wish, you can investigate how well the model would fit the data if the coefficients were rounded to the nearest 10. To do this simply change the formula in cell C4 to '**=90*A4^2-20*A4+100**' and then use fill down to copy this formula to cells C5 to C10.

Or you can plot the graph of a quadratic model that you have found yourself using algebra (as described on Page 4).

If you have time, try modelling the broadband data with a polynomial of **degree 3** i.e. a **cubic** function.

How good is the quadratic model?

The graphs you have drawn should show that the quadratic functions

$$y = \mathbf{90.714}x^2 - \mathbf{17.857}x + \mathbf{100} \quad \text{and} \quad y = \mathbf{91}x^2 - \mathbf{18}x + \mathbf{100}$$

are both reasonably good models of the data for $0 \le x \le 6$.

You can describe how close a model is to the actual data using percentage errors.

$$\textbf{\% error} = \frac{\textbf{predicted value} - \textbf{actual value}}{\textbf{actual value}} \times \mathbf{100}$$

Example

When $x = 2$ (i.e. on 1st January 2002) the Excel model estimates the number of broadband connections to be 370 thousand wheras the model $y = 91x^2 - 18x + 100$ predicts 428 thousand (see Page 5).

$$\% \text{ error} = \frac{428 - 370}{370} \times 100 = 16\%$$

Note the fact that the % error is positive means that the model's prediction is 16% too high, whereas a negative % error would mean the prediction was too low.

Some to try:

1. a) Use a calculator to find the % error when the function $y = 91x^2 - 18x + 100$ is used to model the data for each of the other values of x.

   b) Use a formula in your spreadsheet to find the % error when the function $y = 90.714x^2 - 17.857x + 100$ is used to model the data for each of the other values of x.

2. Write a paragraph interpreting your results from question 1.

3. The models can also be used to estimate what the number of broadband connections were before 2001 using negative values of x.
   a) Use the model $y = 91x^2 - 18x + 100$ to calculate y when $x = -1$,
   b) Explain what information this gives and say whether or not you think it seems realistic.

4. When evaluating a model it is also a good idea to think about how well the function is likely to predict values in the future.
   a) Using $y = 91x^2 - 18x + 100$ calculate the number of broadband connections this model predicts for (i) $x = 10$ (ii) $x = 20$
   b) Comment on your answers to part a).

Teacher Notes

Unit Advanced Level, Working with algebraic and graphical techniques

Notes

This activity shows students how to use Excel to find a quadratic function to model the growth in broadband connections over recent years and also how to check how well a function models data. Students will need to know how to draw and format graphs in Excel before they attempt this. The data is also supplied on an Excel spreadsheet which you can use if you wish to save the time needed to enter the data. (Alternative versions B and C use a graphic calculator and algebra.) The activity ends with some questions about how well the models fit the data and their limitations. These can be done by students as an exrecise or used as a starting point for class discussion. The answers to these questions are given below.

Answers

1 a)

Time (6 mth intervals) x = 0 on 1st January 2001	**No. of Connections (thousands)**	**Model**	**% Error in Model**
x	y	$y = 91x^2 - 18x + 100$	
0	110	100	**– 9%**
1	200	173	**– 14%**
2	370	428	**16%**
3	830	865	**4%**
4	1500	1484	**– 1%**
5	2370	2285	**– 4%**
6	3200	3268	**2%**

b)

Time (6 mth intervals) x = 0 on 1st January 2001	**No. of Connections (thousands)**	**Model**	**% Error in Model**
x	y	$y = 90.714x^2 - 17.857x + 100$	
0	110	100	**– 9%**
1	200	172.857	**– 14%**
2	370	427.142	**15%**
3	830	862.855	**4%**
4	1500	1479.996	**– 1%**
5	2370	2278.565	**– 4%**
6	3200	3258.562	**2%**

2 Main points:
Very little difference between the two models, the unrounded version being just slightly better. Both models better for the larger values of x than for smaller values of x.

3 a) $y = 209$
b) The model estimates that the number of broadband connections on 1st July 2000 was 209 thousand. This is not realistic as it is higher than the number of connections given by the original data for 1st January 2001.

4 a) (i) 9020 thousand (ii) 36 140 thousand
b) Part a) suggests that there will be 9 020 000 broadband connections by January 2006 and 36 140 000 by January 2011. These estimates seem unrealistically high, bearing in mind that there are roughly 60 000 000 people in the UK.